1992 BC 3

a. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t(-\sin t) + e^t \cos t}{e^t \cos t + e^t \sin t}$

$= \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)}$

$= \frac{\cos t - \sin t}{\cos t + \sin t}$

at $t = \frac{\pi}{2}$

$= \frac{\cos\frac{\pi}{2} - \sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + \sin\frac{\pi}{2}} = \frac{0-1}{0+1} = -1$

b, $v = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{[e^t(\cos t - \sin t)]^2 + [e^t(\cos t + \sin t)]^2}$

$= \sqrt{(e^t)^2(\cos^2 t - 2\sin t\cos t + \sin^2 t) + (e^t)^2(\cos^2 t + 2\sin t\cos t + \sin^2 t)}$

$= e^t\sqrt{\cos^2 t + \cos^2 t \cancel{- 2\sin t\cos t} \cancel{+ 2\sin t\cos t} + \sin^2 t + \cos^2 t}$

$= e^t\sqrt{\underbrace{\sin^2 t + \cos^2 t}_{1} + \underbrace{\sin^2 t + \cos^2 t}_{1}}$

$= e^t\sqrt{2}$ at $t = 1$ $= e^1\sqrt{2}$

c, distance $= \int_0^1 v = \int_0^1 e^t\sqrt{2} = e^t\sqrt{2}\Big]_0^1$

$= e^1\sqrt{2} - e^0\sqrt{2}$

$= e\sqrt{2} - \sqrt{2}$

1992 BC 4

a) Left and right hand limits must agree

CONTINUITY

$2(1)-1^2=1$

$1^2+k(1)+p$

$\therefore\ 1=1^2+k+p$

$0=k+p$

DIFFERENTIABLE

Take derivative of both pieces and they should match at $x=1$

$f'=2-2x \quad f'(1)=2-2=0$

$f'=2x+k \quad f'(1)=2+k$

$2+k=0$

$k=-2$

from above $k+p=0$

$\therefore\ -2+p=0$

$p=2$

$k=-2 \quad p=2$

b) The first derivatives are

now $f'=2-2x \leftarrow$ for $x<1$

$f'=2x-2 \leftarrow$ for $x>1$

	1	
$2-2x$	$+\ 0$	~~$-$~~
	$+\ 0$	$-$

	1	
$2x-2$	~~$-$~~	$0\ +$
	$-$	$0\ +$

$\therefore$ f is increasing for all real #'s

c. $f''=-2$ for $x<1$

$f''=2$ for $x>1$

	1	
f''	$-$	$0\ +$

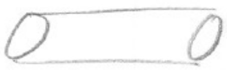
f is continuous at $x=1$.

There is a sign change in f'' at $x=1$ $\therefore$ $(1,1)$ is a point of inflection

note: $2(1)-1^2=1$ original

$1^2+-2(1)+2=1$ $f(x)$ eqn.

1992 BC 5

a. 

$V = \pi r^2 h$

$= \pi \left(\frac{1}{2}\right)^2 32 = 8\pi \text{ in}^3$

b. $\frac{dh}{dt} = 18$

$\frac{dr}{dt} = ?$ $\quad \ell = 1$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left(r^2 \frac{dh}{dt} + h\, 2r \frac{dr}{dt}\right)$

since original ht $= 32$,
one minute later ht $= 32 + 18 = 50$

$V = \pi r^2 h$

$8\pi = \pi r^2 (50)$

stays constant $\frac{8}{50} = r^2$

$\frac{4}{25} = r^2$

$r = \frac{2}{5}$

$\therefore \frac{dV}{dt} = \pi\left(r^2 \frac{dh}{dt} + h\, 2r \frac{dr}{dt}\right)$

$0 = \pi\left(\left(\frac{2}{5}\right)^2 18 + 50(2)\frac{2}{5}\frac{dr}{dt}\right)$

$\frac{dr}{dt} = \frac{9}{125} = .072$

c. $\text{Work} = \int_0^{18} F(x)$

$= \int_0^{18} 2x\, dx$

$= x^2 \Big]_0^{18} = 18^2$

$\therefore$ The work done in the first minute of stretching the chord is 324 inch-pounds.

thus $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ converges

b. If $p=1$

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Integral Test

$$\int_2^{\infty} \frac{1}{n \ln n} = \frac{(\ln n)^2}{2}\Bigg]_2^{\infty}$$

$$= \frac{(\ln \infty)^2}{2} \quad \frac{(\ln 2)^2}{2} - \infty$$

$\therefore$ Diverges

Since $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ DIVERGES in part b

$\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ diverges by

the comparison test